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جامعة بيرزيت
BIRZEIT UNIVERSITY

Mathematics Department

Math 330

Final Exam 1st.Semester 08/09

Student name: ...... ID no.: sec........

Q# 1(68Points)

1- Given the Taylor polynomial expansion

$$\cos(h) = 1 - \frac{h^2}{2} + 0(h^4)$$

$$\sin(h) = 1 - \frac{h^3}{3!} + 0(h^5)$$

then the order of approximation of cosh sinh is

(a) $0(h^9)$ (b) $0(h^4)$ (c) $0(h^5)$ (d) $0(h^{20})$ (e) None of the above

$\cosh \sinh = \left(1-\frac{h^2}{2}\right)(o(h^5)) + \left(1-\frac{h^3}{3!}\right)o(h^4) + o(h^4)\,o(h^5) + \left(1-\frac{h^2}{2}\right)\left(1-\frac{h^3}{3!}\right)$

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$= \cancel{o(h^5)} + o(h^4) + o(h^9) + 1 - \frac{h^3}{3!} - \frac{h^2}{2} + \frac{h^5}{12}$

$= 1 - \frac{h^2}{2} - \frac{h^3}{3!} + o(h^4).$

2- The fixed points of $g(x) = 4x - \frac{x^2}{2} - 4$ are $x = 2, x = 4$ one of the followings is true

(a) Both of them are attractive fixed points.
(b) Both of them are repulsive fixed points.
(c) $x = 2$ is repulsive fixed point but $x = 4$ is attractive.
(d) $x = 4$ is repulsive fixed point but $x = 2$ is repulsive.
(e) None of the above.

$g'(x) = 4 - x$

4

$|g'(2)| = 4 - 2 = 2 > 1$ fail

$|g'(4)| = |4-4| = 0 < 1$ ✓ fixed

3- The order of convergence R and the multiplicity M of the root $p=1$ of $f(x)=x^3-3x^2+3x-1$ are

(a) $R=2,\quad M=3$

(b) $R=1,\quad M=3$

(c) $R=2,\quad M=1$

(d) ~~$R=1$,~~ $M=2$

(e) None of the above.

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$f(1)=0$

$f'(1)=3x^2-6x+3$

$f''(x)=6x-6,\ f''(1)=0$

$f'''(x)=6$

4- Given the data $(1,0.5000),(2,0.3333),(3,0.250)$. An estimation of $f(2.5)$ using polynomial interpolation and 6-digits arthematics is

(a) 0.401248 (b) 0.281225 (c) 0.0937250 (d) 0.218725 (e) None

$P_2(x)=y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}+y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}+y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

2

$=0.050000\frac{(x-2)(x-3)}{(-1)(-2)}+0.3333\frac{(x-1)(x-3)}{(1)(-1)}+0.250\frac{(x-1)(x-2)}{(3-1)(3-2)}$

$=0.025(x^2-5x+6)-0.3333(x^2-4x+3)-0.125(x^2-3x+2)$

$-6.25\times10^{-3}+0.249975-0.109375=0.149975$

5- An upper bound for the error in the above estimation when $f(x)=\frac{1}{x+1}$ is

(a) $2x10^{-3}$ (b) $4x10^{-2}$ (c) $1x10^{-4}$ (d) $1x10^{-5}$ (e) None

2 $|E_2(x)|\le\frac{h^3M_3}{9\sqrt3}=\frac{(1)^3(0.375)}{9\sqrt3}$

$=0.024$

$(x+1)^{-1}$

$f'(x)=-1(x+1)^{-2}$

$f''(x)=2(x+1)^{-3}$

$f'''(x)=-6(x+1)^{-4}$

$=\frac{-6}{(x+1)^4}$

$=\frac{-6}{(2)^4}=0.375$

6- Given the data $(1, f(1)), (1.1, f(1.1)), (1.2, f(1.2)), f(x) = \frac{1}{x+1}$. An estimation of $0(h^2)$ of $f'(1)$ is

(a) 4.29655 (b) 14.7512 (c)-0.20489 (d)0.2489 (e)None

4

$-2 + 1.4286 -$

$f'(1) = \frac{f(1.1) - f(1)}{0.1}$

$f'(x) = \frac{-4f_0 + 3f_1 - f_2}{2h} = \frac{-4f(1) + 3(f(1.1)) - f(1.2)}{2(0.1)}$

$= \frac{f(1.1) - f(1)}{0.1}$

$= \frac{-4(0.5) + 3(1.4285) - 0.455}{0.2}$

7- The cost of solving $Ax = b$ when A is 3x3 matrix using Cramer's method is

(a)59 (b)56 (c) 45 (d) 42 (e)None

4

$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$

$x_1 = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{A}$, $x_2 = \dots$, $x_3 = \dots$

$= 4 \times 3 = 12 + 2 = 14$

$4 \times 14 = 56 + 3 = 59$

8- When estimating the integral $\int_{-1}^{1} \frac{e^x}{x} dx$ using Gauss-Legender 2 points formula, the answer is

~~(a) 2.1126 (b)8.0226 (c)3.2514 (d)4.1121~~ (e) None

4

$G_2(f) = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{e^{-1/\sqrt{3}}}{-1/\sqrt{3}} + \frac{e^{1/\sqrt{3}}}{1/\sqrt{3}} = -0.972 + 3.085$

$= 2.112$

9- One of the following statements is true:

(1) The bisection method is costly but it always converges.
(2) Newton method is the fastest and cheapest method.
(3) False position method is as fast as the secant method
(4) The secant method is faster than bisection but slower than Newton.

10- The best upper bound of the number of iteration needed when using the bisection method to estimate a root of $f(x)$ in $[2,5]$ with error less than 10^{-6} is

(a) 20 (b) 21 (c) 19 (d) 22 (e) None of the above

$$\left|\frac{5-2}{2^{n+1}}\right| \leq 10^{-6}$$

$$\frac{3}{2^{n+1}} \leq 10^{-6} \Rightarrow \frac{2^{n+1}}{3} > \frac{1}{10^{-6}} \Rightarrow 2^{n+1} > 3000000$$

$$(n+1)\ln 2 > \ln 3000000$$
$$(n+1)\ln 2 > 14.9$$
$$n+1 > 21.5 \quad n > 20.5 \quad n=21$$

11- When using Newton method to estimate the root of $f(x)=x^3-\cos x-2$ in $[1,2]$, $P_0=1.75$ the 1^{st} Iteration P_1 is

(a) 1.5 (b) 1.4 (c) 1.7 (d) 1.8 (e) None of the above

$$f'(x) = 3x^2+\sin x.$$

$$P_{k+1} = g(x) = 1.75 - \frac{x^3-\cos x-2}{3x^2+\sin x} = 1.75 - \frac{3.54}{10.17}$$

$$P_{k+1} = P_k - \frac{f(P_k)}{f'(P_k)} = P_k - \frac{(P_k)^3-\cos(P_k)-2}{3(P_k)^2+\sin(P_k)}$$

12- When using the false position method to estimate the root of $f(x) = x^3 - \cos x - 2$ in[1,2] then c_1 (the 2^{nd} iteration)is

(a) 1.2696 (b) 1.8215 (c) 1.1235 (d) 1.72134 (e) None of the above

4y

$C_{n+1} = b_n - \frac{f(b_n)(b_n - a_n)}{f(b_n) - f(a_n)} = 2 - \frac{6.416}{6.416 - (-1.54)}$

$f(1) = -1.54$

$f(2) = 6.42$

$= 1.194$

$f(1.194) = -0.167$

$b_1 = ~~6.42~~ 2$

$a_1 = 1.194$

$C_2 = 2 - \frac{6.42(0.806)}{6.42 - (-0.167)} = 1.269$

13- Using the secant method to estimate the root of $f(x) = x^3 - \cos x - 2$ in[1,2] and $P_0 = 1, P_1 = 2$ then P_3 is

(a) 1.1235 (b) ~~1.2696~~ (c) 1.72134 (d) 1.8215 (e) None of the above

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$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)} = 2 - \frac{6.42(1)}{6.42 - (-1.54)} = 1.192$

$P_3 = 1.192 - \frac{(-0.678)(1.192 - 2)}{(-0.678) - (6.42)}$

$= 1.269$

14- When using the Gauss-Sidel method to estimate the solution of

$g_1(x,y,z) = (2x - 3y + 2z - 1)/7$

$g_2(x,y,z) = (3x - 2y - z + 2)/5$

$g_3(x,y,z) = (x - y + 3z - 3)/2$

$and P_0 = (1,2,3)$

Then if $P_1 = (p_1, q_1, r_1), r_1$ is

(a) 0.14286
(b) 2.7286
(c) 0.68572
(d) 2.5
(e) None of the above

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$P_1 = (2(1) - 3(2) + 2(3) - 1)/7$
$= 0.143$

$q_1 = 3(0.143) - 2(2) - 3 + 2)/5$
$= -0.914$

$r_1 = \frac{0.143 - (-0.914) + 3(3) - 3}{2}$
$= 3.5285$

15-The order of the error of the formula is

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

(a) $0(h^3)$ (b) $0(h^2)$ (c) $0(h)$, (d) $0(h^4)$ (e) None of the above

4

$f(x+h) = f(x) + h f'(x) + \frac{h^2 f''(x)}{2!}$

$f'(x) = f(x+h) - f(x) - \frac{h f''(c)}{2!}$

16- The constants c_0, c_1 and x_1 so that the quadration formula

$\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$ has the highest possible degreeof precision are

(a) $c_0 = \frac{5}{9}, c_1 = \frac{8}{9}$ $x_1 = \frac{5}{9}$ (b) $c_0 = \frac{1}{4}, c_1 = \frac{2}{3}, x_1 = \frac{3}{4}$ (c) $c_0 = \frac{1}{4}, c_1 = \frac{3}{4}, x_1 = \frac{2}{4}$

(d) $c_0 = \frac{3}{4}, c_1 = \frac{2}{3}, x_1 = \frac{1}{4}$ (e) None of the above

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$\int_0^1 dx = 1$, $Q[1] = C_0 + C_1 \Rightarrow C_0 + C_1 = 1$,

$\int_0^1 x\,dx = \frac{x^2}{2}\Big]_0^1 = \frac{1}{2}$, $Q[x] = C_1 X_1 \Rightarrow C_1 X_1 = 1/2$

$\int_0^1 x^2 dx = \frac{x^3}{3} = \frac{1}{3}$, $Q[x^2] = C_1 X_1^2 = 1/3$

$\Longrightarrow \frac{C_1 X_1^2}{C_1 X_1} = \frac{1/3}{1/2} \Rightarrow X_1 = \cancel{\frac{2}{3}}$, $\Rightarrow C_1 = \cancel{\frac{1/2}{2/3}} = \frac{3}{4}$.

$C_0 = \cancel{\frac{1}{4}}$

17- For the above formula, the error constant k

(a) $\frac{1}{135}$ (b) $\frac{1}{90}$ (c) $\frac{1}{15750}$ (d) $\frac{1}{216}$ (e) None of the above

4

$\cancel{\int X^3 dx = \frac{1}{3} \quad Q[x^2] =}$

$\int x^3 dx = \frac{x^4}{4} = \frac{1}{4}$. $Q[x^3] = C_1 (x_1^3) = \frac{3}{4}\left(\frac{2}{3}\right)^3 = \frac{2}{9}$

$\frac{1}{4} - \frac{2}{9} = k f^{(3)}(c)$, $f(x) = x^3$, $f^{(3)}(x) = 3!$

$\frac{1}{36} = 3!\,k \Rightarrow k = \frac{1}{36 \times 6} = \frac{1}{216}$

Q#2(16Points) Estimates $\int_1^2 e^{-x/2}\,dx$, $h = 0.25$ one of the followings is true

(a) Using composite trapezoidal rule.
(b) Using composite Simpson rule.
(c) Find an upper bound for the errors in a and b.
(d) Compare the above estimations to the exact answer.

$h = 0.25$

x	1	1.25	1.5	1.75	2
f(x)	0.6065	0.535	0.473	0.417	0.37

(a) $T(f,h) = \frac{h}{2}\left[f(1) + 2f(1.25) + 2f(1.5) + 2f(1.75) + f(1)\right]$

$= \frac{0.25}{2}\left[0.6060 + 2(0.535) + \cdots + 0.37\right]$

$= 0.4783$

(b) $S_{1/3}(f,h) = \frac{h}{3}\left[f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + f(2)\right]$

$= \frac{0.25}{3}\left[0.6066 + 4(0.535 + 0.417) + 2(0.473) + 0.37\right]$

$= 0.4775$

فرع (c)، (d) ناقصة

$\cancel{\int_1^2 e^{-x/2} = -\frac{1}{2}e^{-x/2}\Big]_1^2 = -\frac{1}{2}\left[e^{-1/2} - e^{-1}\right] = -0.1193}$

$E_T(f,h) = \cancel{\frac{f h^3 f''(c)}{12}}$

$\cancel{|E_T(f,h)| \le \frac{h^3 M_2}{12}}$ $M = \max f''(c)$ $f'(x) = -\frac{1}{2}e^{-x/2}$

$f''(x) = \frac{1}{4}e^{-x/2}$

≤ 0.2

$f''(1) = 0.15$